Recall

Definite Integral

$$\int_a^b f(x)\,dx = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} f(x_k^*)\Delta x_k$$

$$\text{Area} = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k^*)\Delta x_k$$

$f(x)$ is the rate of change of area function $A(x)$.

6.6 Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is

Any antiderivative of $f(x)$.

I.O.W. $F(x) = \int f(x)\,dx$

Ex:

① find $\int x^2 dx$

$$\int_0^1 x^2\,dx = \frac{x^3}{3} + C$$

② evaluate between 0 & 1

$$\left(\frac{x^3}{3} + C\right)\Big|_0^1 =$$

$$\left(\frac{1^3}{3} + C\right) - \left(\frac{0^3}{3} + C\right) = \frac{1}{3}$$

Recall * Area - in a question is always positive

* "signed area" - the definite integral of a function that is negative on an interval will be negative.

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

* $\int_a^b f(x)\,dx = F(b) - F(a)$ where $F(x)$ is any antiderivative of $f(x)$.

i.e. $\int_a^b f(x)\,dx = \left[\int f(x)\,dx \right] \Big|_a^b$

* $\int_a^b f(x)\,dx = - \int_b^a f(x)\,dx$..

☆ is there any difference between

$$\int_a^b f(x)\,dx \quad \text{and} \quad \int_a^b f(t)\,dt\,?$$

exactly $f(x) = x^2+1$ and $f(t) = t^2+1$?

☆ simplified area idea

Mean Value Theorem for integrals.

f continuous $\Rightarrow$ $\exists$ c in $[a,b]$
there exists
such that
$$\int_a^b f(x)\,dx = f(c)\,[b-a]$$

$\Longrightarrow$ preview of coming attractions.
the average value of a function
will be $\frac{1}{b-a}\int_a^b f(x)\,dx$

Bob: semi-definite integral

$$\int_a^{x^2} f(t)\,dt$$

$$\frac{d}{dx}\int_2^x x\,dx = \frac{d}{dx}\left(\frac{x^2}{2}\Big|_2^x\right)$$

$$\frac{d}{dx}\left(\frac{x^2}{2} - 2\right)$$

$$= x$$

$$\frac{d}{dx}\int_2^{x^2} t\,dt = \frac{d}{dx}\left(\frac{x^2}{2}\Bigg|_2^{x^2}\right)$$

$$= \frac{d}{dx}\left(\frac{x^4}{2} - \frac{4}{2}\right) = \frac{4x^3}{2} = 2x^3$$

$$\frac{d}{dx}$$

$$x^2(2x)$$

$$\frac{d}{dx}\left[\int_2^{x^2} t\,dt\right] =$$

$$t\Big|_{t=x^2}\,\frac{d}{dx}(x^2)$$

$$(x^2)(2x)$$

$$\frac{d}{dx}\int_a^{g(x)} f(x)\,dx = \frac{d}{dx}\left(\int_a^{x} f(x)\,dx\right)\Big|_{x=g(x)} \cdot g'(x)$$

$$f(x)\Big|_{g(x)} \cdot g'(x)$$

outside: $\int_a^x f(x)\,dx$

inside: $g(x)$

$$\frac{d}{dx}\left(r(s(x))\right) = r'(s(x)) \cdot s'(x) = f(g(x)) \cdot g'(x)$$

$f(x) = x^2$ $\qquad \frac{x^3}{3} - 0$

$\frac{d}{dx} = 2x$

$\frac{d}{dx} \int_0^x x^2 \, dx = x^2$

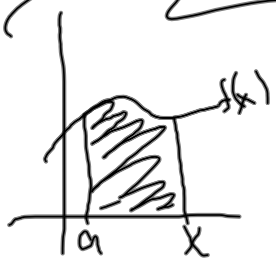
6.6/49 $\frac{d}{dx}\left(\int_1^x \sin(\sqrt{t})\,dt\right)$ (b) $\frac{d}{dx}\int_0^x e^{t^2}\,dt$

$\sin(\sqrt{x})$ e^{x^2}

Recall $\int_a^b f(x)\,dx = \lim_{\substack{\max\Delta x \\ \to 0}} \sum_{k=1}^{n} f(x_k^*)\,\Delta x_k$ Definite Integral 6-5

and Area $= \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)(\Delta x_k)$ 6.4 def^n of area

and



f(x) is the derivative of the area function A(x) [which describes the area under f(x) from a to x] 6.2 & 6.1

I.e. ≡ Id est

"that is"

e.g ≡ exempli gratia

for example...

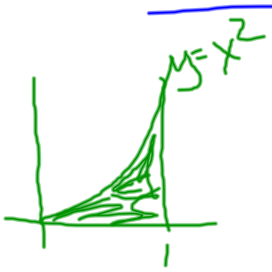
6.6 Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is ANY antiderivative of $f(x)$.

I.O.W.

$$F(x) = \int f(x)\,dx$$



$$\int_0^1 x^2\,dx =$$

① find an antiderivative

$$\int x^2\,dx = \frac{x^3}{3} + c$$

② evaluate between 0 & 1

$$\left(\frac{x^3}{3}+c\right)\Bigg|_0^1 = \left(\frac{1^3}{3}+c\right) - \left(\frac{0^3}{3}+c\right)$$

$$= \frac{1}{3}$$

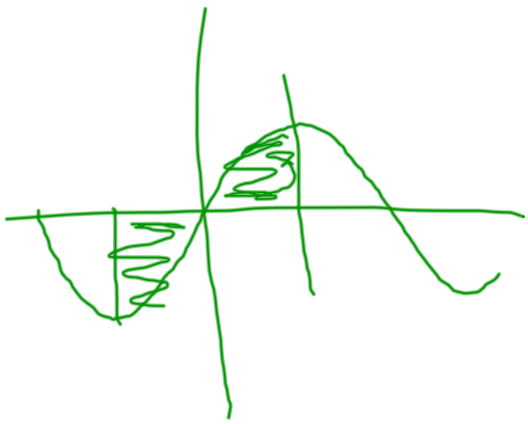
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x \, dx$$

① $\int \sin x \, dx = -\cos x + C$

② $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x \, dx = \left(-\cos x\right)\Big|_{-\frac{\pi}{2}}^{\pi/2}$

$= -\cos\left(\frac{\pi}{2}\right) - \left(-\cos\left(\frac{-\pi}{2}\right)\right)$

$= -0 + 0 = 0$



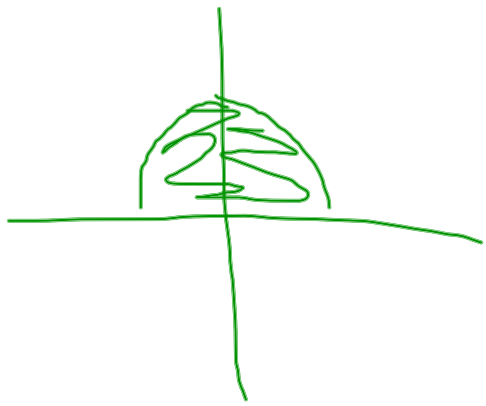
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx$$

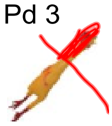
① $\int \cos x \, dx = \sin x + C$

② $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx = \left(\sin x\right)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

$= \left(\sin \frac{\pi}{2}\right) - \left(\sin\left(\frac{-\pi}{2}\right)\right)$

$= 1 - (-1) = 2$





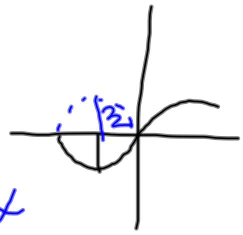
$$\int_0^{\frac{\pi}{2}} \sin x \, dx$$

$$= (-\cos x)\Big|_0^{\frac{\pi}{2}}$$

$$= \left(-\cos\left(\frac{\pi}{2}\right)\right) - (-\cos(0))$$

$$0 + 1 = 1$$

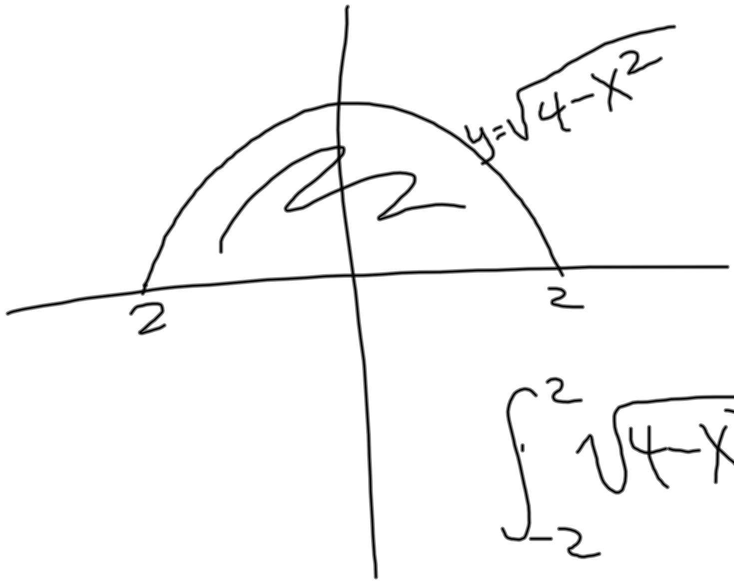
$$\int_{-\frac{\pi}{2}}^{0} |\sin x| \, dx$$



$$= \int_{-\frac{\pi}{2}}^{0} -\sin x \, dx$$

$$= (\cos x)\Big|_{-\frac{\pi}{2}}^{0} =$$

$$(\cos 0) - \left(\cos\left(-\frac{\pi}{2}\right)\right) = 1 - 0 = 1$$



$$\int_{-2}^{2} \sqrt{4-x^2}\,dx$$

Recall

* Area – always positive
* the definite integral of a function (over an interval) where the values of that function are negative is a negative number.

$\boxed{\int_a^b f'(x)\,dx = f(b) - f(a)}$

* $\int_a^b f(x)\,dx = F(b) - F(a)$ where $F(x)$ is any antiderivative

i.e. $\int_a^b f(x)\,dx = \left[\int f(x)\,dx\right]_a^b$

* $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

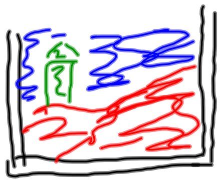
* is there any difference between

$$\int_a^b f(x)\,dx \text{ and } \int_a^b f(t)\,dt?$$

basically the same question as

"are $f(x)=x^2+1$ and $f(t)=t^2+1$ the same f^n"

* simplified area idea

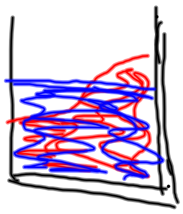


Mean Value Theorem for integrals

f continuous $\Rightarrow \exists$ c in $[a,b]$

there exists

such that $$\int_a^b f(t)\,dt = f(c)[b-a]$$



preview of coming attractions

we are going to define the average value of a function

$$= \frac{1}{b-a}\int_a^b f(x)\,dx$$

Bob: semi-definite integral

$$\int_a^x f(t)\,dt = F(x) - F(a)$$

where $F(*)$ is any antiderivative of $f(x)$. i.e. $F(x) = \int f(x)\,dx$

$$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = F'(x) - \frac{d}{dx}(F(a))$$

$$= f(x)$$

$$\int_2^x t\,dt = \left(\frac{t^2}{2}\right)\Big|_2^x = \frac{x^2}{2} - \frac{2^2}{2}$$

$$= \frac{x^2}{2} - 2$$

$$\frac{d}{dx}\left(\int_2^x t\,dt\right) = \frac{d}{dx}\left(\frac{x^2}{2} - 2\right) = x$$

$$\frac{d}{dx}\int_2^{x^2} t\,dt = \frac{d}{dx}\left(\frac{t^2}{2}\right)\Big|_2^{x^2} = \frac{d}{dx}\left(\frac{(x^2)^2}{2} - \frac{2^2}{2}\right)$$

$$x \mapsto x^2 \mapsto \int_2^u t\,dt$$

$$= \frac{d}{dx}\left(\frac{x^4}{2} - 2\right)$$

Composi

$$= 2x^3$$

$$\frac{d}{dx}\left(\int_2^{x^2} t\,dt\right) = (x^2)\frac{d}{dx}(x^2) = x^2(2x)$$

$$= 2x^3$$

$$\frac{d}{dx}\left(\int_2^u t\,dt\right)\Big|_{u=x^2} \cdot \frac{d}{dx}(x^2)$$

6.6 examples 2010-11-29 Pd 3

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